Inhomogeneous state in nonequilibrium superconductor/normal-metal tunnel structures: A Larkin-Ovchinnikov-Fulde-Ferrell-like phase for nonmagnetic systems

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We analyze nonequilibrium states in a tunnel superconductor-normal-metal (N/S/N) structure in the presence of a tunnel current I. We use an approximation of an effective temperature T and calculate the current-voltage I-V characteristics. It is shown that the I-V dependence may have an S-shaped form. We determine nonuniform current $I(x)$ and temperature $T(x)$ distributions that arise as a result of instability of the uniform state with negative differential conductance ($dI/dV < 0$). We discuss an analogy with equilibrium superconductors with an exchange field in which nonuniform states predicted by Larkin/Ovchinnikov and Fulde/Ferrell are possible.

I. INTRODUCTION

Study of superconductor/normal metal (S/N) or superconductor/superconductor heterostructures has been a popular topic during last decades.[1–4] Both the equilibrium and nonequilibrium properties of such systems were under intensive investigation. Nonequilibrium phenomena are of special interest because they are essentially different from the equilibrium ones but several interesting effects have been observed even in the cases when a deviation from the equilibrium is small. As an example, one can mention a nonmonotonic behavior of the resistance of S/N structures as a function of temperature T or applied voltage V.[5–9]

If the applied voltage is not too small, the superconductor S (for example, in a N/S/N system) goes out of the equilibrium and new phenomena come into play. Study of nonequilibrium effects in superconducting heterostructures as well as in homogeneous superconductors in the presence of optical or microwaves radiations has been carried out during a long time (see reviews[10–12] and references therein). For example, Eliashberg[13] suggested to stimulate superconductivity by a microwave radiation that leads to an essential deviation of the distribution function f from the equilibrium one. As the energy gap Δ in S is related to the function f via the self-consistency equation, the energy gap and the critical temperature may increase in the presence of a microwave radiation, which results in an increase in the critical current.[14] Another interesting effect caused by a nonequilibrium distribution function is the sign reversal of the Josephson critical current j_c in a multiterminal SNS junction.[15] It turns out that, provided the distribution function in the normal-metal N controlled by additional N′ electrodes differs significantly from the equilibrium one, the critical current j_c becomes negative (π state).[16–18]

In recent years, the interest in studies of nonequilibrium effects in S/N or S/S′ heterostructures has revived. This is due to the progress in nanotechnology, the possible applications of such structures in low-temperature devices[19] and to the progress in theoretical research.[1–4,20]

Perhaps, the simplest system in which one can study nonequilibrium effects is a N/S/N structure with a bias current I_b. Such a structure was studied theoretically in recent papers.[21,22] In Ref. 21 it was assumed that the resistance of the S/N interfaces is negligible. The calculated I-V characteristics (CVC) were shown to be of the so-called N-shape type, that is, three values of voltages correspond to one value of the bias current I_b. Earlier this type of the I-V characteristics was studied in other superconducting systems.[12,23–25]

The opposite case of a large S/N interface resistance was considered in Ref. 22. In this case the I-V curve is of the S-type, i.e., three values of the current correspond to one value of the voltage V between the N leads. Such kind of the CVC was known from previous studies of the superconducting structures. For example, it can be realized in S/I/S junctions[10,11,26,27] and in granular superconductors.[28,29] The authors of Ref. 22 investigated the stability of this system and came to the conclusion that the system is stable. However, this conclusion is valid only in the case of small lateral dimensions of the structure. From a general theory of the system with negative differential conductance ($G_d = dI/dV < 0$), it is known that the states with negative G_d are unstable and of the S-shape I-V curve a stratification of the current density occurs as a result of the instability.[30,31] Inhomogeneous states in different superconducting systems were studied in approximate models with account for electron-electron or electron-phonon inelastic scattering.[10,11]

In the present paper, we analyze the N/S/N structure of the type considered in Ref. 22 assuming however that lateral dimensions are large. The latter assumption turns out to be very important. We find in this limit that a new inhomogeneous state is possible, which contrasts the situation in small sized systems. Physics of this inhomogeneous state is very similar to that of the inhomogeneous state in equilibrium superconductors with an exchange field (the Larkin-Ovchinnikov-Fulde-Ferrell states[32,33]). It is worth emphasizing though that, in the system considered here, there are no magnetic interactions and their role is played by a finite voltage V (see below). Under certain assumptions we calculate the I-V curve and study possible inhomogeneous states. Most of the results are obtained in an analytical form. In particular, we find the spatial distribution of the current density $j(x)$ and energy gap $\Delta(x)$ and draw an analogy with those in the LOFF states.
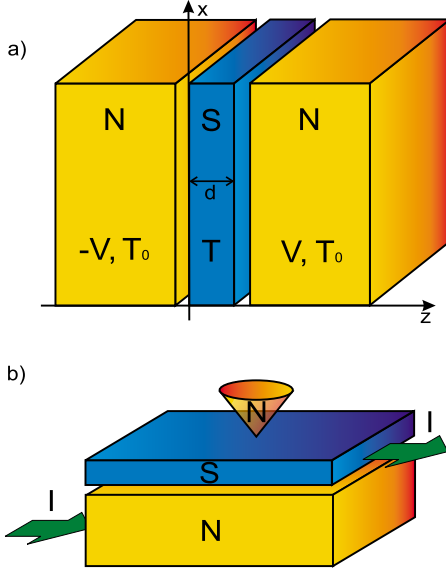
FIG. 1. (Color online) (a) The system under consideration. (b) The proposed system in which an inhomogeneous distribution of the energy gap can be probed with a point-contact spectroscopy.

II. MODEL. BASIC EQUATIONS

In order to calculate physical quantities we use a kinetic equation for the nonequilibrium distribution functions $f_\pm$. The procedure of the derivation is quite standard (see, e.g., Ref. 34) and is based on using quasiclassical Keldysh matrix Green functions $\hat{g}$. They are expressed in terms of the two distribution functions $f_\pm$ as $\hat{g} = \hat{g}^R(\hat{\tau}_3 f_- + \hat{\tau}_0 f_+) - (\hat{\tau}_3 f_- + \hat{\tau}_0 f_+)\hat{g}^A$, where $\hat{\tau}_3$ is the $\hat{\tau}_z$ Pauli matrix and $\hat{\tau}_0$ is the unit matrix. The functions f_+ and f_- correspond to the functions f_L and f_T introduced by Schmid and Schön.[35]

The considered N/S/N system is shown in Fig. 1(a). We assume that the tunneling probability through the interface is small and that the retarded (advanced) Green's functions $\hat{g}^{R(A)}$ have the same form as in a bulk superconductor: $\hat{g}^{R(A)} = g^{R(A)}\hat{\tau}_3 + f^{R(A)}i\hat{\tau}_2$ with $f^{R(A)} = \Delta/\sqrt{(\epsilon \pm i\gamma)^2 - \Delta^2}$ and $g^{R(A)} = \sqrt{1 - (f^{R(A)})^2}$. The only difference is that, in contrast to the bulk case, the damping γ, caused by proximity effect, enters the formula for the function $f^{R(A)}$. In this limit the damping γ is equal to $\gamma \approx \epsilon_0$, where $\epsilon_0 = D/(R_{SN}\sigma d)$, $D = v^2\tau/3$ is the diffusion coefficient, R_{SN} is the SN interface resistance per unit area, σ and d are the conductivity (in the normal state) and the thickness of the superconductor. The thickness of the S layer is assumed to be small ($d \ll \xi_S \approx \sqrt{D/\Delta}$) so that all quantities do not depend on the z coordinate. The opposite case was considered in Refs. 21 and 36. The thickness of the N leads is supposed to be large ($d_N \gg \xi_S$) so that the N metals are in the equilibrium state.

The function $f_-(\epsilon, V)$ appears due to a branch imbalance and, being dependent on the electric potential V, determines the current I. In case of a fully symmetric N/S/N system $f_- = 0$. The function $f_+(\epsilon, V)$ determines the energy gap Δ. Since the function $f_+(\epsilon, V)$ depends on the applied voltage V, the gap Δ is also a function of V. The dependence $\Delta(V)$ is given by the self-consistency equation

$$1 = \lambda \, \text{Re} \int_0^{\theta_D} d\epsilon \frac{f_+(\epsilon, V)}{\sqrt{\tilde{\epsilon}^2 - \Delta^2}}, \quad (1)$$

where $\tilde{\epsilon} = (\epsilon + i\gamma)$ and λ and θ_D are the coupling constant and Debye energy, respectively. Equation (1) is a generalization of the conventional BCS equation to the nonequilibrium case.

The functions $f_\pm$ can be found from the kinetic equation that can be written in S in the form (see, for example, Ref. 37)

$$\frac{\partial[\mathcal{N}_S(\epsilon)f_\pm]}{\partial t} - \frac{\partial[\mathcal{D}_\pm(\epsilon)\,\partial f_\pm]}{\partial x^2} = \sum_{\alpha=l,r} \epsilon_\alpha \mathcal{A}_{\alpha\pm}(\epsilon) + S_{in}\{f_\pm\}, \quad (2)$$

where

$$\mathcal{N}_S(\epsilon) = \text{Re}\{g^R(\epsilon)\} = \text{Re}\frac{\tilde{\epsilon}}{\sqrt{\tilde{\epsilon}^2 - \Delta^2}} \quad (3)$$

is the normalized density of states (DOS), $\epsilon_{l,r} = (D/R_{l,r}\sigma d)$ with $R_{l,r}$ being the resistance of the left (right) S/N interfaces and $\mathcal{D}_\pm$ is an energy-dependent diffusion coefficient,

$$\mathcal{D}_\pm(\epsilon) = \frac{D}{2}\left[1 + \frac{\tilde{\epsilon}\tilde{\epsilon}^* \mp \Delta^2}{\sqrt{\tilde{\epsilon}^2 - \Delta^2}\sqrt{\tilde{\epsilon}^{*2} - \Delta^2}}\right] \quad (4)$$

with $\tilde{\epsilon}^* = \epsilon - i\gamma$.

The functions $\mathcal{A}_{l,r\pm}(\epsilon)$ in Eq. (2) are defined as

$$\mathcal{A}_{l,r\pm}(\epsilon) = \mathcal{N}_S(\epsilon)[F_\pm(\epsilon, V_{l,r}) - f_\pm(\epsilon)]. \quad (5)$$

The distribution functions in the normal metals are assumed to have the equilibrium forms shifted by the applied voltages V. In a fully symmetric system, the voltages in the right (V_r) and left (V_l) N metals are equal to $V_r = -V_l \equiv V$. In this situation, the functions $F_\pm$ entering Eq. (5) take the simple form

$$F_\pm = [\tanh(\epsilon + eV)\beta_0 \pm \tanh(\epsilon - eV)\beta_0]/2, \quad (6)$$

where $1/2\beta_0 = T_0$ is the reservoir temperature. In the limit of the weak coupling between N and S layers the density of states in the N metals is supposed to be unperturbed by the proximity effect: $\mathcal{N}_N(\epsilon) = 1$.

The first terms on the right-hand side of the kinetic Eq. (2) describe the tunneling of quasiparticles from (to) the normal electrodes, whereas the last one is an inelastic collision term. It consists of the electron-electron and electron-phonon collision terms ($S_{in} = S_{e-e} + S_{e-ph}$) and is on the order of $f_\pm(\epsilon)/\tau_{e-e}$ and $f_\pm(\epsilon)/\tau_{e-ph}$, where τ_{e-e} and τ_{e-ph} are the electron-electron and electron-phonon inelastic-scattering times. Equation (2) contains the DOS, $N_S(\epsilon, \Delta)$, depending on the energy gap $\Delta(V)$. Therefore Eqs. (1)–(6) are coupled and the problem of finding the distribution function f_+ is not easy.

III. MODERATE INTERFACE RESISTANCE. HOMOGENEOUS CASE

We consider first a homogeneous stationary state assuming that all the quantities do not depend on the coordinate x.

However, the problem of solving Eqs. (1)–(6) is too complicated even under this assumption and therefore we consider only two limiting cases.

First, we analyze the limit of a very weak electron-electron and electron-phonon interactions: $\Delta \gg \epsilon_{l,r} \gg \tau_{e-e}^{-1}$, τ_{e-ph}^{-1}, when the collision terms in Eq. (2) can be neglected. In other words, the S/N interface resistance R_{SN} is not too high: $R_{SN}\sigma d \ll \{D\tau_{e-e}, D\tau_{e-ph}\}$. This approximation has been used in many papers (see, for instance, Refs. 20 and 22). For a symmetric N/S/N system we use Eq. (6) and obtain from Eq. (2) the following solutions:

$$f_+ = F_+, \quad f_- = 0. \tag{7}$$

Substituting the function f_+ into Eq. (1) and shifting the energy in the integral by eV in the first term and by $-eV$ in the second one, we reduce Eq. (1) to the following form:

$$1 = \frac{\lambda}{2}\text{Re}\int_{-\theta_D}^{\theta_D}\frac{\tanh(\epsilon\beta_0)}{\sqrt{(\epsilon - eV + i\gamma)^2 - \Delta^2}}d\epsilon. \tag{8}$$

Remarkably, the form of Eq. (8) is the same as the one for a superconductor in the presence of an exchange field. In other words, the problem of the nonequilibrium superconductor in the N/S/N system is to a great extent equivalent to the problem of the equilibrium superconductor with the distribution function $f_+=\tanh(\epsilon\beta_0)$ in the presence of an "exchange" field eV.

The latter problem has been attracting a lot of attention since the pioneering works by Larkin and Ovchinnikov[33] and Fulde and Ferrell.[32] For the model with the exchange field, these authors have predicted a new state called now LOFF state. They considered a clean superconductor with the exchange field h (or a strong magnetic field) acting on spins of electrons. They demonstrated that at zero temperature the energy gap Δ remained unchanged unless the exchange energy h exceeded the value Δ_0, where Δ_0 is the energy gap at zero temperature in the absence of h.

However, in addition to this solution of the self-consistency equation, there is another, unstable, solution for Δ: $\Delta(h)=\Delta_0\sqrt{h^2/h_c^2-1}$ with $h_c^2=\Delta_0^2/2$ and $h_c\leq h\leq\Delta_0$. Therefore, in the interval $h_c\leq h\leq\Delta_0$ there are three possible solutions for Δ: 0, $\Delta(h)$, and Δ_0 (the trivial solution $\Delta=0$ always exists). The situation resembles the behavior of a nonideal gas described by the van der Waals equation of state and thus, one can expect a stratification of the electron system. Indeed, as it was shown in Refs. 32 and 33, an inhomogeneous state with the energy gap $\Delta(r)$ varying in space turns out to be more favorable than the homogeneous one.

Using the equivalence between the N/S/N system at a finite voltage V and the equilibrium superconductor in the presence of the exchange field one may expect an inhomogeneous LOFF-like state in the system considered here. A multivalued (in a certain region of parameters) dependence of the energy gap Δ on the applied voltage V and damping γ has recently been found for an N/S/N system in Ref. 22. The function $\Delta(V)$ had a form similar to that determined by Eq. (8). It was established that at some values of the parameters the CVC had the so-called S-shaped form. This type of the

CVC is well known in bulk superconductors and has already been obtained both theoretically (see the review[10,11]) and experimentally[26,27] several decades ago.

Although, one could expect an inhomogeneous state in the N/S/N systems, the authors of Ref. 22 came to the conclusion that the homogeneous state had to remain stable. This statement is correct for the superconductor island of a small size considered in Ref. 22 but cannot remain valid for large superconducting films sandwiched between normal metals. Note also that the authors of Ref. 22 studied the stability of the system in a short-time interval: $t\lesssim\hbar/\Delta$ (a similar dynamic behavior of the order parameter in a collisionless superconducting system was studied earlier in Ref. 38). On the other hand the instability develops on much longer characteristic times $t\sim\hbar/\epsilon_0$.

In this case, the states corresponding to the part of the CVC with negative differential resistance are unstable and the stratification of the current density occurs in the system (see the review[31]). This structure resembles the LOFF coordinate dependence of the order parameter and of other physical quantities. However, in contrast to the LOFF state in the equilibrium superconductors with the exchange field, the coordinate dependence cannot be found minimizing the free energy because we consider a system out of the equilibrium.

IV. HIGH INTERFACE RESISTANCE. NONHOMOGENEOUS CASE

Although the similarity between the problem involved and the problem of LOFF is most clearly seen in the limit, $\epsilon_{l,r} \gg \tau_{e-e}^{-1}$, τ_{e-ph}^{-1}, one has to solve in this limit complicated integrodifferential Eqs. (1) and (2) and this can be done only numerically. The problem can be solved analytically in another limit,

$$\tau_{e-ph}^{-1} \ll \epsilon_{l,r} \ll \tau_{e-e}^{-1}. \tag{9}$$

In this limit the electron-phonon interaction is very weak and the electron-phonon collision term in Eq. (2) can be neglected. The largest term in Eq. (2) is the electron-electron collision term $S_{e-e}\{f_\pm\}$.

Note that the electron-electron τ_{e-e}^{-1} and electron-phonon τ_{e-ph}^{-1} scattering rates depend on energy ϵ (Ref. 39) so that Eq. (9) can be satisfied not at all energies. If the mean-free path is not too short ($s/T < l$ but $l<v/3\Delta$ because we consider the dirty limit), these scattering rates are: $\tau_{e-e}^{-1} \sim \gamma_{e-e}(T_e)\epsilon^2/T_e^2$ and $\tau_{e-ph}^{-1} \sim \gamma_{e-ph}(T_e)\epsilon^3/T_e^3$ with $\gamma_{e-e} = \pi\lambda_{e-e}T_e^2/(8\hbar E_F)$ and $\gamma_{e-ph}=\pi\lambda_{e-ph}T_e^3/(2\hbar s^2 p_F^2)$, where λ_{e-e} and λ_{e-ph} are electron-electron (electron-phonon) interaction constants, E_F is the Fermi energy, and s is the sound velocity (see, for example, Ref. 34). The characteristic energy ϵ_{ch} in our case is on the order $\epsilon_{ch}\sim T_e\sim T_c$; thus, for the Al S films the inelastic-scattering rates are: $\tau_{e-e}^{-1}\approx10^8$ s^{-1} and $\tau_{e-ph}^{-1}\approx10^6$ s^{-1}.[40] At temperatures lower than T_c, the interval between τ_{e-e}^{-1} and τ_{e-ph}^{-1} becomes larger because τ_{e-ph}^{-1} has a stronger dependence on ϵ than τ_{e-e}^{-1}. However, one has to take into account the dependence of τ_{e-e}^{-1} and τ_{e-ph}^{-1} on $\Delta(T)$. Therefore, if the frequency $\epsilon_0/\hbar=D/(R_{SN}\sigma d)$, which is determined by the S/N interface resistance R_{SN}, is chosen in the interval 10^6–10^8 s^{-1}, the approximation of an effective temperature can be applied to the case of these films.

It is worth noting that the violation of the condition (9) leads only to breakdown of the effective temperature approximation but not to disappearance of the effects under consideration. For these effects it is important only to have the S-shape CVC, which is realized, for example, also in the case of the frequency $\epsilon_0/\hbar$ large in comparison with τ_{e-e}^{-1} and τ_{e-ph}^{-1} (see the preceding section and Ref. 22). However the approximation of the effective temperature allows one to solve the problem in an analytical form. In the limit of high impurity concentration $(l < s/T)$, the inelastic electron-phonon-scattering rate τ_{e-ph}^{-1} depends on energy even stronger,[29,41–43] that is, the window between τ_{e-e}^{-1} and τ_{e-ph}^{-1} becomes larger and the situation is even more favorable for applicability of our approach. If the frequency $\epsilon_0/\hbar$ is less than τ_{e-ph}^{-1}, then the heat absorbed by quasiparticles in the superconductor is released mainly to the lattice, but not to the normal electrodes, and our approach is not applicable. However, one can show that in this case the CVC of the system also has the S-shape form and all the effects considered here remain qualitatively unchanged, although the analytical approach is not possible in this case.

In the limit determined by Eq. (9) the electron-phonon interaction is very weak and the electron-phonon collision term in Eq. (2) can be neglected. The largest term in Eq. (2) is the electron-electron collision term $S_{e-e}\{f_{\pm}\}$. Due to a high rate of the electron-electron collisions one can assume that the distribution function f_+ has an equilibrium form, $f_+(\epsilon) = \tanh(\epsilon\beta)$, with an effective temperature $T \equiv (2\beta)^{-1}$ depending on the coordinate x. Perhaps for the first time, the approximation of the effective temperature has been introduced in the study of nonequilibrium superconductors by Parker[44] (see also Refs. 29, 31, and 45). In order to find this temperature, we multiply Eq. (2) by ϵ, drop the first term on the left-hand side and integrate the equation over all energies. This procedure leads to an equation describing the energy conservation. The contribution of the collision term $S_{e-e}\{f_+\}$ equals zero because this term conserves the total energy and we obtain the equation for T

$$-\frac{\partial(\mathcal{M}(\widetilde{T})\,\partial\widetilde{T}/\partial x)}{\partial x} = \sum_{\alpha=l,r}\kappa_\alpha^2[\mathcal{J}(V_\alpha,\widetilde{T}) - \mathcal{S}(\widetilde{T})], \quad (10)$$

where $\kappa_{l,r}^2 = (D/R_{l,r}\sigma d)$ and $\widetilde{T} \equiv T/T_0$ is the normalized temperature, and

$$\mathcal{M}(\widetilde{T}) = (\widetilde{T}/D)\int_0^\infty dy\,y^2 \mathcal{D}_+(y,\Delta\beta)\cosh^{-2}(y).$$

where the function $\mathcal{D}_+(y,\Delta\beta)$ is defined in Eq. (4).

The functions $\mathcal{J}(V_\alpha,T)$ and $\mathcal{S}(T)$ are the heat source (Joule heat) and drain due to the tunneling of electrons into the N electrodes. They can be written as

$$\mathcal{J}(V_a,\widetilde{T}) = \int_0^\infty dy\,y\mathcal{N}_S(y)\{\tanh(y) - [\tanh(y_+) + \tanh(y_-)]/2\},$$

$$(11)$$

$$\mathcal{S}(\widetilde{T}) = \int_0^\infty dy\,y\mathcal{N}_S(y)[\tanh(y) - \tanh(y/\widetilde{T})], \quad (12)$$

where $y = \epsilon\beta_0$, $y_\pm = y \pm eV\beta_0$, and $\widetilde{T} = T/T_0$.

In order to solve Eq. (10) we consider for simplicity the symmetric case when $V_r = -V_l \equiv V$. In the homogeneous case the dependence of the effective temperature on the applied voltage can be found from the balance equation: $\mathcal{J}(V,\widetilde{T}) = \mathcal{S}(\widetilde{T})$. Explicit expressions can be derived analytically in the limits of low and high temperatures.

In the limit of high temperatures, $V\beta_0 \ll 1$, we come to the following formula:

$$\widetilde{T} - 1 \equiv (T - T_0)/T_0 \approx (V\beta_0)^2 \quad (13)$$

whereas at low temperatures, $V\beta_0 \gg 1$, $|eV-\Delta| \gg T_0$ we obtain

$$\widetilde{T} - 1 \approx eV/(\Delta - eV). \quad (14)$$

Note that at low temperatures T_0 both the terms J and S are, as they should be, exponentially small but the effective temperature T is not. Making these estimations, we assume that $\gamma \to 0$. Thus, it is not obvious that the effective temperature T and energy gap Δ will be multivalued functions of V as it takes place in another limit considered in the previous section. Numerical calculations of the integrals [Eqs. (11) and (12)] show that the situation qualitatively remains un-
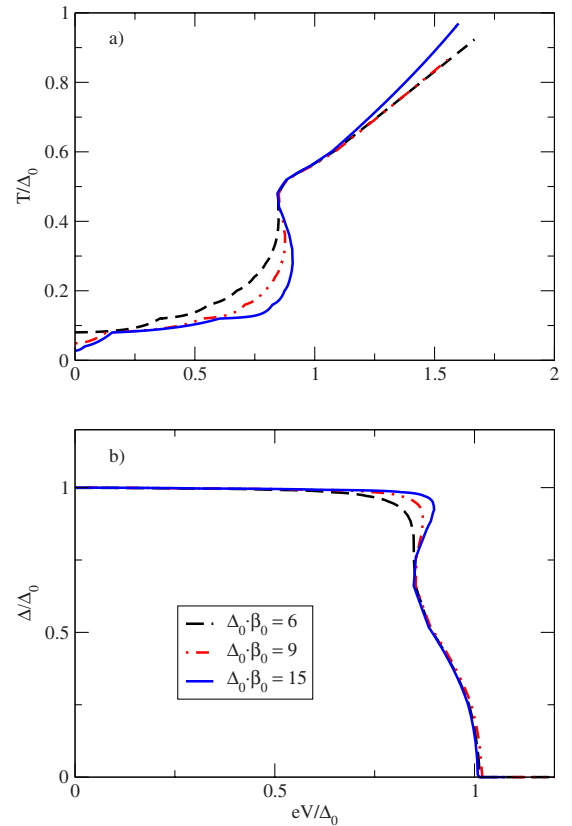

FIG. 2. (Color online) (a) Normalized effective temperature and (b) energy gap as functions of the applied voltage V for $\gamma/\Delta_0 = 0.01$ and different temperatures $T_0 = 1/(2\beta_0)$ (shown in the inset).
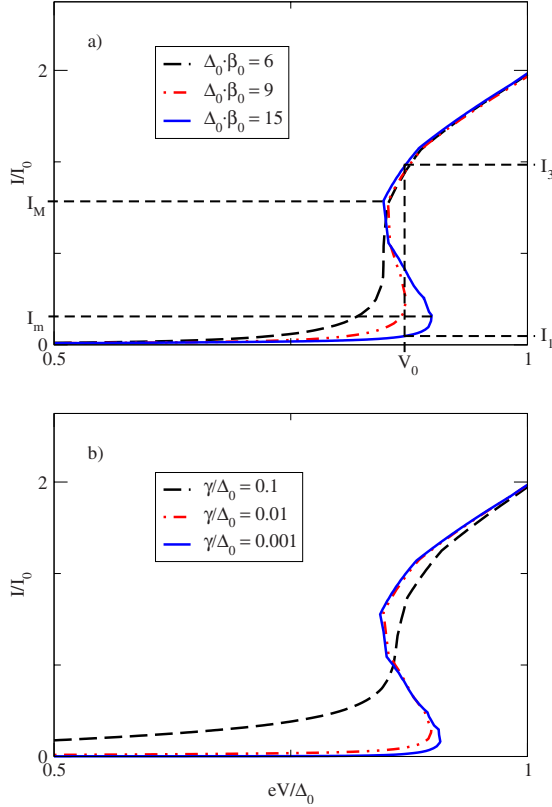
FIG. 3. (Color online) (a) The CVC of the system in the homogeneous state for $\gamma/\Delta_0 = 0.01$ and different temperatures. The part of the CVC corresponding to an inhomogeneous state (a broad high-temperature domain) is shown by the dashed vertical line. (b) The CVC in the homogeneous state for different damping γ. The temperature T_0 is chosen so that $\Delta_0\beta_0 = 15$.

changed, i.e., the quantities T and Δ have three values in a certain interval of voltages V. The dependencies $T(V)$ and $\Delta(V)$ are shown in Figs. 2(a) and 2(b). One can see that in a narrow interval of the voltage, the effective temperature can have three values (T_1, T_2, T_3) and therefore the energy gap Δ is also a multivalued function of V, which is analogous to the behavior in superconductors with an exchange field h.

The current through the system is given by the formula

$$I = \frac{1}{eR_b}\int_0^\infty d\epsilon \mathcal{N}_S[\epsilon, \Delta(T)]F_-(\epsilon, V), \qquad (15)$$

where $F_- = [\tanh(\epsilon + eV)\beta_0 - \tanh(\epsilon - eV)\beta_0]/2$. The CVC obtained from Eq. (15) is shown in Fig. 3 for different temperatures T_0 [Fig. 3(a)] and damping γ [Fig. 3(b)].

It is clearly seen from Fig. 3 that the CVC has an "S-shaped" form. This type of the CVC may also occur in semiconductors[31] as a result of the so-called overheating mechanism and in bulk superconductors in the presence of a dissipative current.[10,11,26,29] This phenomenon is possible for a special dependence of the heat source $J(V,T)$ and heat absorption term $S(T)$ on the effective temperature T and applied voltage V. The "N-shaped" CVC may also arise as a result of the overheating mechanism.[12,21,23–25]
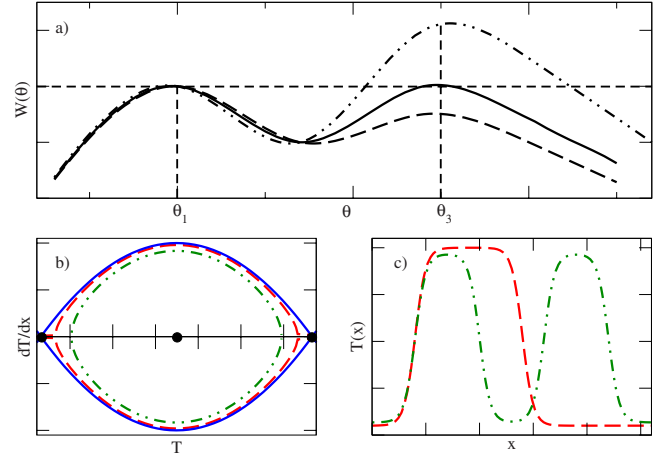

FIG. 4. (Color online) (a) The potential $W(\theta)$ for different V: $V = V_0$ (solid line); $V < V_0$ (dashed line), and $V > V_0$ (dashed-dotted line). (b) Phase trajectories $(T, dT/dx)$. Solid line shows a separatrix [a domain wall in $T(x)$ distribution]; dashed and dotted lines show domainlike structure of $T(x)$ distribution; (c) spatial dependence of the effective temperature corresponding to a domain-like structure (arbitrary units).

One can show in the same way as it was done in Ref. 31 that the states corresponding to the part of the CVC with a negative differential resistance are unstable and, in the case of a fixed total current,[46] the system is stratified into layers with different effective temperatures and current densities. The form of the current filaments can be found from Eq. (10).

We introduce a new effective "temperature" $\vartheta = \int_1^{\widetilde{T}} d\widetilde{T}_1 M(\widetilde{T}_1)\widetilde{T}_1$ and a function of this temperature $W(\vartheta) = \int_0^\vartheta \{J[V, \widetilde{T}(\vartheta_1)] - S[\widetilde{T}(\vartheta_1)]\}d\vartheta_1$. Integrating Eq. (10) over the temperature $\widetilde{T}$ and then over ϑ we arrive at the equation

$$(1/2)l_0^2(\partial\vartheta/\partial x)^2 = W_0 - W(\vartheta, V), \qquad (16)$$

where $l_0^{-1} = \kappa_r = \kappa_l$.

Equation (16) is already quite simple and its solution ϑ describes the dependence of the temperature $\widetilde{T}$ on the coordinate x along the interface. One can rather easily see that this dependence can be nontrivial. This originates from the fact that in a narrow interval of V, where the CVC is a multivalued function, the function $W(\vartheta, V)$ has three extrema at $\vartheta_k(T_k)$, where $k = 1, 2, 3$ [see Fig. 4(a)]. As a consequence, the solutions of Eq. (16), $l_0(\partial\vartheta/\partial x) = \sqrt{2}\sqrt{W_0 - W(\vartheta, V)}$, have different forms: solitons (instantons), oscillatory temperature distributions, or domain walls.

In Fig. 4(b) we show qualitatively phase trajectories illustrating this conclusion. At a certain voltage V_0 the function $W(\vartheta)$ has the same values at the maxima: $W(\vartheta_1) = W(\vartheta_3)$. This means that the voltage V_0 satisfies the condition: $\int_{\vartheta_1}^{\vartheta_3}\{J[V_0, \widetilde{T}(\theta)] - S[\widetilde{T}(\theta)]\}d\theta = 0$. The trajectory connecting these maxima (separatrix) corresponds to a solution like a domain wall: $l_0(\partial\vartheta/\partial x) = \sqrt{2}\sqrt{W(\vartheta_1, V_0) - W(\vartheta, V_0)}$. The spatial distribution of the effective temperature T related to this trajectory is a decay of T from the value T_3 to T_1 over a length on the order of l_0. The trajectories close to the sepa-

ratrix describe the domain structure [see Fig. 4(c)]. This structure is analogous to the one in a superconductor with an exchange field in the LOFF state.

At the same time, the inhomogeneous state in the SN systems out of the equilibrium differs from the LOFF state. In the latter case the solution of the self-consistency equation should correspond to the minimum of the free energy whereas in our case the choice of the solution is dictated by the bias current I_b and by the stability against small perturbations. One can show that the most stable solution has the minimal number of zeros of the function $\partial T(x)/\partial x$.[31,47]

This fact suggests the following scenario: (a) with increasing the bias current I_b the dependence $I(V)$ follows the CVC for the homogeneous case. (b) At $I_b > I_m$ the voltage V jumps to a lower value close to V_0 and remains almost constant with increasing I_b [see Fig. 3(a) and Ref. 31]. (c) When the current I_b exceeds the value I_3, the voltage V increases along the CVC for the homogeneous case. If I_b decreases from I_3, the voltage decreases and a jump to a higher voltage close to V_0 occurs at $I_b \lesssim I_M$ (a hysteretic behavior).

It would be interesting to study experimentally the inhomogeneous distribution of the effective temperature $T(x)$, current, and the gap $\Delta(x)$. This could be done, for example, by applying the point-contact spectroscopy to N/S systems [see a possible setup in Fig. 1(b)]. The distribution function f_+ in these systems has the same form as in N/S/N systems. Therefore, by measuring the spatial distribution $\Delta(x)$, one can get information about the inhomogeneous states in the N/S systems.

It is interesting to note that the ideas that the symmetry breaking of the LOFF type may occur not only in the superconductors but also in the quantum chromodynamics, astrophysics, and in cold gases (see the review[48] and references therein) have been put forward recently. Many theoretical papers have been published since the pioneering works[32,33] in which different aspects of the LOFF state in superconductors and conditions necessary for this state were analyzed (see

Refs. 48 and 49 and references therein). As to the experimental observation of the inhomogeneous states predicted, the situation is not as clear. Although some observations in low-dimensional superconductors and superconductors with heavy fermions can be interpreted in terms of the LOFF states, there are no convincing evidences in favor of such states in ordinary s-wave superconductors.

So, the investigation of nonequilibrium superconductors, for example, superconductors in tunnel N/S/N systems, can provide one more possibility to observe the LOFF state experimentally. Such experiments may be useful for understanding the LOFF states and the conditions under which they can be realized. We mention here another interesting example of the analogy between a nonequilibrium superconducting system and equilibrium superconducting system with an exchange field, namely, the π state may arise not only in S/F/S junctions[50–52] but also in S/N/S junctions with a non-equilibrium distribution function[15–18] (here F denotes a ferromagnetic layer).

V. CONCLUSIONS

In conclusion, using the approximation of an effective temperature of quasiparticles, we have studied nonequilibrium states in a tunnel NSN structure with a low barrier transparency. It is found that for certain values of parameters, the CVC of the system may have an S-shaped form. The uniform state corresponding to the part of the CVC with negative differential conductance is unstable and therefore a nonuniform current $I(x)$ and temperature $T(x)$ distribution is established in the system with a fixed total current. We discuss the analogy with the nonuniform LOFF states in equilibrium superconductors and the possibilities of experimental observation of the nonuniform states in SN structures.

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